LESSON SUMMARY

CXC CSEC MATHEMATICS

UNIT SIX:
Measurement

Lesson
11

Measuring Shapes

Textbook: Mathematics, A Complete Course by Raymond Toolsie, Volume 1

(Some helpful exercises and page numbers are given throughout the lesson, e.g. Ex 4j page 130)

INTRODUCTION

Measurement is a useful tool in solving problems involving length, area and volume. In this lesson we will look at determining perimeter, area and volume of simple shapes and combination of these shapes.

OBJECTIVES

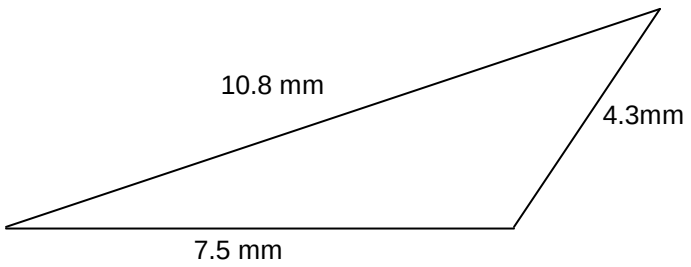
At the end of this lesson you will be able to:

a) calculate the perimeter of a polygon, a circle, and combinations of these shapes;
b) calculate the length of an arc of a circle;
c) calculate the area enclosed by plane shapes and combinations of these shapes;
d) calculate the area of a sector of a circle;
e) calculate the surface area and volume of a simple prism, a pyramid, and a sphere.

6.1 Perimeter of Plane Figures.

Perimeter is the distance around a shape or figure. The perimeter can be found by adding the length of each side that makes up the shape.

Example: Calculate the perimeter of the following triangle.



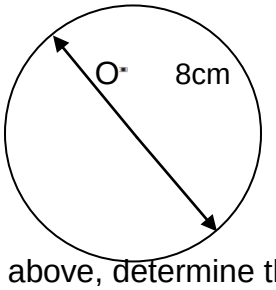
Solution: the perimeter $P = 10.8\ mm + 7.5\ mm + 4.3mm$

$= 22.6mm$.

Circles

The distance around a circle is called the circumference of the circle. The circumference of a circle $C = diameter \times \pi$.

Example:



Using the diagram above, determine the circumference of the circle. Take π as 3.14.(Ex 4j page 130)

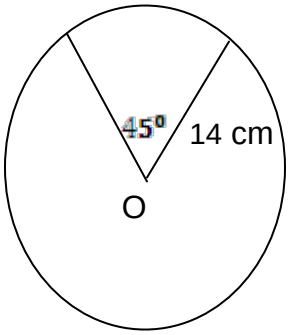
Solution: $C = 8cm \times 3.14$

$= 25.1cm$.

The length of an arc of a circle

The length of an arc is a fraction or part of the circumference. To determine the length of an arc, multiply the circumference of the circle by the fraction that best represents the arc.

Example: .(Ex 4j page 130)



The previous diagram shows a circle of radius 14 cm with a sector angle 45^0 .

Calculate the length of the minor arc. Take π as $\frac{22}{7}$.

Solution: the length of the minor arc is $\frac{45}{360} = \frac{1}{8}$ of the circumference. Therefore the length of the minor arc is:

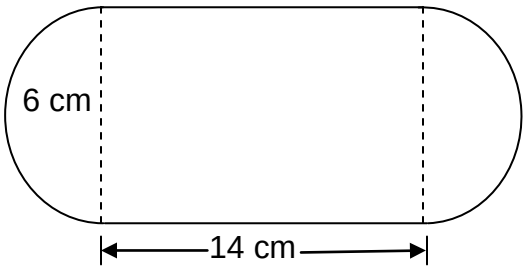
$\frac{1}{8} \times 28cm \times \frac{22}{7}$

$= 11cm.$

Compound shapes

Figures may be formed from a combination of simpler shapes. The length of the sides may be stated clearly or you may have to determine the length from other information given.

Example: Determine the perimeter of the following model of a race track (Ex 4k page 134):



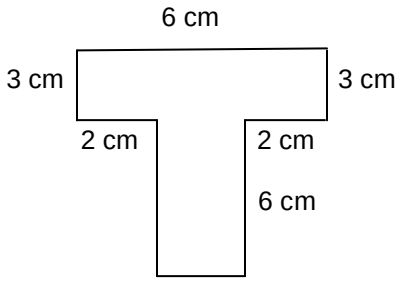
Solution: This shape is formed from a rectangle and two semi-circles. The length of the rectangle is 14cm and its width is 6cm. Note only the lengths of the rectangle are part of the perimeter of the figure. The width of the rectangle is the diameter of the semi-circle. This is needed to find the circumferences of the semi-circles which are part of the perimeter. The semi-circles are identical therefore to find the perimeter of the model just determine the circumference of a circle with a diameter of 6 cm and add it to the lengths of the rectangle.

The perimeter of the model race track $(6\ cm\ \times 3.14) + (2 \times 14\ cm)$

$18.84 + 28$

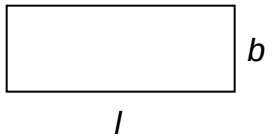
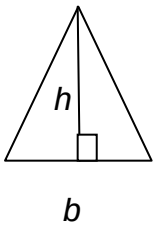
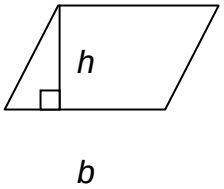
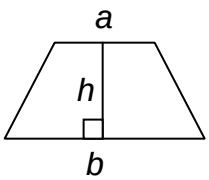
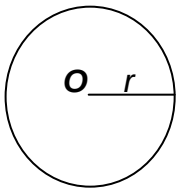
$= 46.84\ cm$

In the figure shown; determine the perimeter. (Ex 4h page 121)

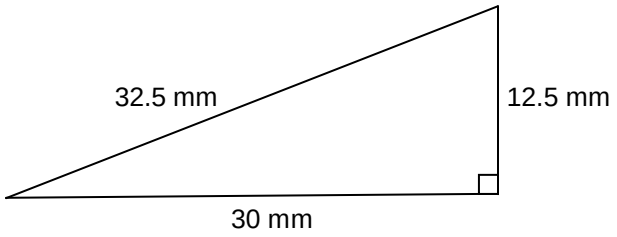


6.2 Area of plane figures

The area of a plane or flat figure is the size of the space within the shape. To find the area of a figure may be as simple as applying a formula. The following table provides some formulae for finding the area of flat shapes.

Name	Shape	Formula
rectangle		$A = lb$
triangle		$A = \frac{1}{2}bh$
parallelogram		$A = bh$
trapezium		$A = \frac{1}{2}(a + b)h$
circle		$A = \pi r^2$

Example: Calculate the area of the following triangle. (Ex 4g page 118)



Solution: This involves applying the formula. The angle between the height and the base is always 90° therefore if we take the base as 30 mm then the height is 12.5 mm.

Thus the area

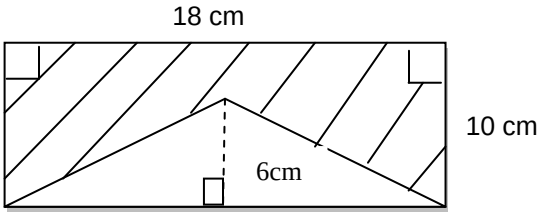
$$A = \frac{1}{2} \times 30\ mm \times 12.5\ mm$$

$$= 187.5\ mm^2.$$

Area of compound figures

The area of a compound figure can be found by finding the area of each shape that makes up the figure and adding these areas to get the total area of the figure. Sometimes however you may have to subtract to find the required area.

Example:



Calculate the area of the shaded figure above. (Ex 4h page 121)

Solution: To determine the shaded area we have to find the area of the triangle and subtract it from the area of the rectangle.

$$Area\ of\ rectangle = 18\ cm \times 10\ cm$$

$$= 180\ cm^2$$

$$Area\ of\ triangle = \frac{1}{2} \times 18\ cm\ \times 6\ cm$$

$$= 54\ cm^2.$$

$$Area\ of\ shaded\ region = 180\ cm^2 - 54\ cm^2$$

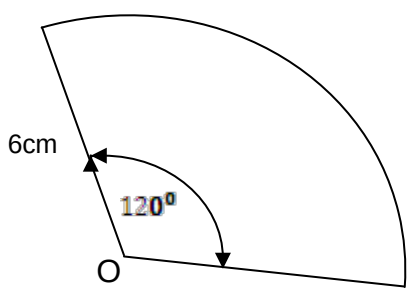
$$= 126\ cm^2.$$

Area of a sector

The area of a sector is part the area of a circle with the same radius as the radius of the circle. The formula for radius of a sector is therefore:

$\frac{\theta}{360} \times \pi r^2$, where θ is the sector angle.

Example: Evaluate the area of the following shape. (Ex 4j page 130)



Use π as 3.14 and state your answer correct to three significant figures.

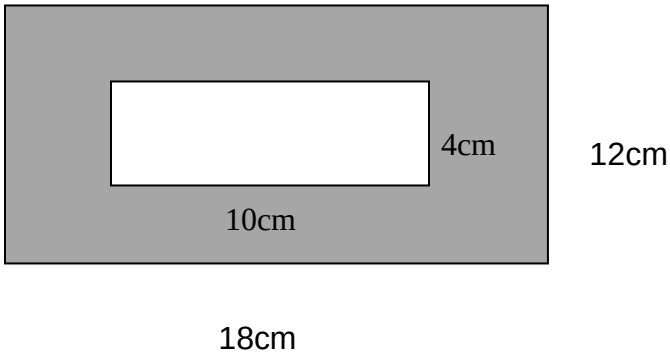
Solution:

$$\frac{120}{360} \times 3.14 \times (6cm)^2$$

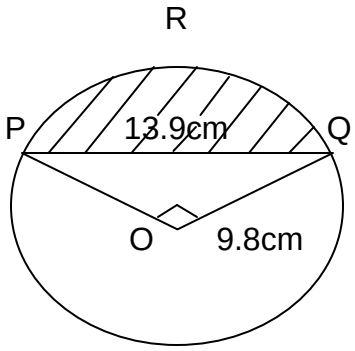
$$= 37.7cm^2$$

ACTIVITY 2

1. Evaluate the area that is shaded in the following figure. (Ex 4h page 121)



2.

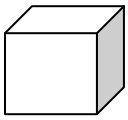


Determine the area of the shaded region PQR. Use π as $\frac{22}{7}$ and state your answers correct to one decimal place.

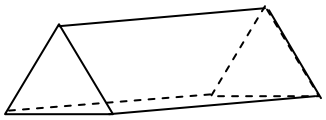
6.3 Surface of Solids

Total Surface Area of a simple prism, a pyramid, and a sphere

Examples of prisms:



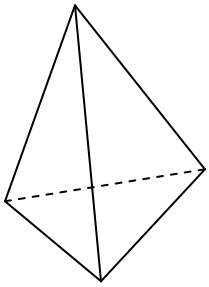
Cube



Triangular prism

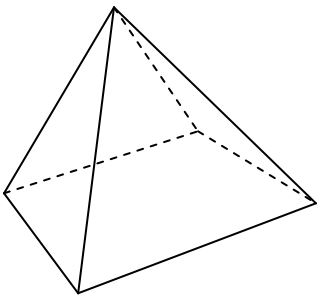
Examples of pyramids:

Apex



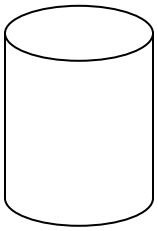
Tetrahedron

Apex

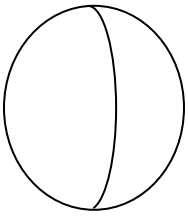


Rectangular base

Other solids:



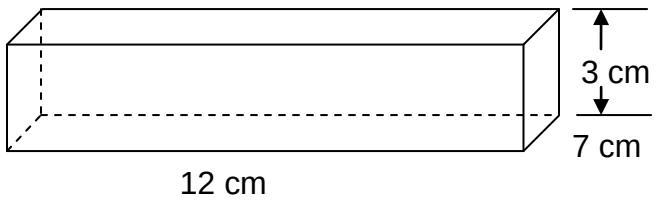
Cylinder



Sphere

The total surface area of a prism and a pyramid can be found by finding the area of each face and adding these areas.

Example:



Calculate the surface area of the uniform solid shown above, with the stated measurements. (Ex 4m page 141)

Solution: We have to find the area of 3 pairs of rectangles. Therefore the total surface area:

$$= 2(7 \times 3) + 2(3 \times 12) + 2(12 \times 7)$$

$$= 42 + 72 + 168$$

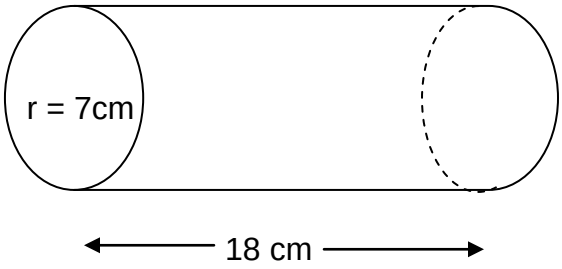
$$= 282\ cm^2$$

Total surface area of a cylinder and a sphere

Finding the surface area of a cylinder involves finding the area of the two circular cross sections (which are the same) and adding this to the curved surface area.

Example:

Calculate the total surface area of the following uniform solid:



Use π as 3.14.

Solution:

The area of two circular cross sections is $2 \times 3.14 \times 49cm^2$

$= 307.7cm^2$

The curved surface area is the area of a rectangle whose length is the height of the cylinder and whose width is the circumference of the circular cross section.

Circumference of the circular cross section $= 2 \times 7cm \times 3.14$

$= 43.96cm$

Curved surface area is $43.96cm \times 18cm$

$= 791.3cm^2$

Total surface area $= 307.7cm^2 + 791.3cm^2$

$= 1099cm^2$

The surface area of a sphere can be found using the formula $4\pi r^2$

Example:

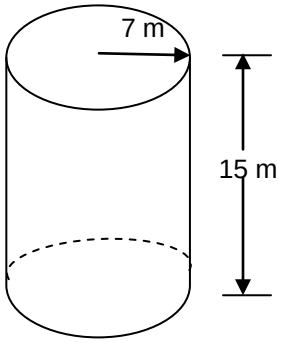
Evaluate the surface area of a sphere of radius 8.5 cm. Use π as 3.14.(Ex 4p page 153)

Solution: By simply applying the formula we have the surface area

$= 4 \times 3.14 \times (8.5cm)^2$

$= 907.5cm^2$

ACTIVITY 3

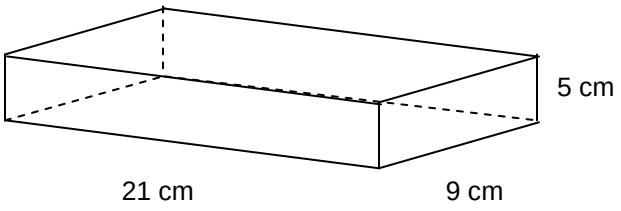


The figure above represents a closed cylindrical tank of height 15 m. The radius of the cross-section is 7m calculate the curved surface area of the tank. (Ex 4n page 142)

6.4 Volume of Solids

The volume (or capacity) of many uniform solids can be found by multiplying the area of its base by its height or by applying a simple formula.

Example 1: Calculate the volume of the following cuboid. (Ex 4m page 141)



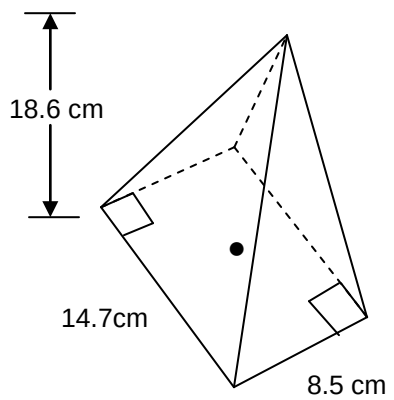
Solution: The area of the base is $21\ cm \times 9\ cm$

$= 189\ cm^2$

If we take the height as 5 cm then the volume of the cuboid $= 189\ cm^2 \times 5\ cm$

$= 945 cm^3$.

Example 2:



The diagram above shows a rectangular-based pyramid of altitude 18.6 cm and dimensions 8.5 cm by 14.7 cm. Calculate the volume of the pyramid. (Ex 4o page 149)

Solution:

The formula for the volume of a pyramid is $V = \frac{1}{3}Ah.$ A is the area of the base and h is the altitude or perpendicular height of the pyramid.

The Area of the rectangle base is $14.7cm \times 8.5cm$

$= 124.95\ cm^2$

Therefore the volume $= \frac{1}{3(124.95\ cm^2 \times 18.6\ cm)}$

$= \frac{1}{3} \times 2324.07$

$= 774.7cm^3$

ACTIVITY 4

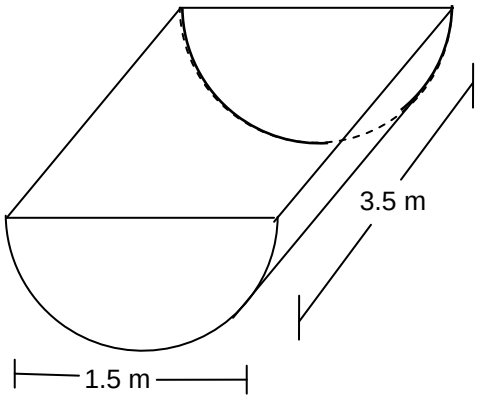
1. Calculate the volume of a concrete block measuring 38 cm by 18 cm by 14 cm.

2. Calculate the volume of the sphere of diameter 17 cm. Use $\pi = 3.14$.

ASSESSMENT

CXC question

1.



[Take $\pi = 3.142$]

[Curved surface area of a cylinder = 2 πrh]

The diagram above, **not drawn to scale,** represents an open metal container. The cross-section of the container is a semi-circle of diameter 1.5 m. The length of the container is 3.5 m.

(a) Write down the radius of the cross-section of the container.

(b) Calculate, in m^2, to two decimal places

(i) the area of the cross-section of the container

(ii) the outer curved surface area of the container

(iii) the total outer area of the container

(c) Calculate the capacity, in m^3, of the container.

(d) Water is poured into the container at a rate of 30 litres per minute. Calculate the length of time, in minutes it would take to just fill the container.

[$[1m]^3 = 1000l$]

Conclusion

The concepts presented here have many real life applications. The student is encouraged to look for these applications and practice them. Geometry also consists of many concepts that can be applied in real life situations. In the lesson that follows we will look at some concepts that are related to lines and angles.